

SUPERFLUID NEUTRON STARS

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Neutron stars are believed to contain (neutron and proton) superfluids. I will give a summary of a macroscopic description of the interior of neutron stars, in a formulation which is general relativistic. I will also present recent results on the oscillations of neutron stars, with superfluidity explicitly taken into account, which leads in particular to the existence of a new class of modes.

1 Superfluid relativistic hydrodynamics

Neutrons stars are believed to contain in their interior superfluid neutrons and superconducting protons. This is suggested by nuclear physics calculations and by the very long relaxation time scales after 'glitches' (sudden increases of the neutron star angular velocity). Over the last years, we have been developing a formalism that can describe the relativistic hydrodynamics of mixtures of superfluid or superconductors and possibly ordinary fluids ^{1,2,3}.

The hydrodynamics of several species labelled by the index X , with respective particle currents n_X^ρ and respective electric charge per particle e^X , can be described by the Lagrangian

$$\mathcal{L} = \Lambda_M + \sum_X e^X n_X^\rho A_\rho + \frac{1}{16\pi} F_{\rho\sigma} F^{\sigma\rho}, \quad (1)$$

where A_ρ is the electromagnetic gauge form and $F_{\rho\sigma}$ the corresponding electromagnetic tensor, and where Λ_M is the 'hydrodynamical' part of the Lagrangian, depending only on the particle currents n_X^ρ , the variations of which define the momentum covectors: $\delta\Lambda_M = \mu_X^\rho \delta n_X^\rho$. This variational principle, corresponding to perfectly conducting fluids, leads to matter conservation equations, one for each species, $\nabla_\rho n_X^\rho = 0$ (one can generalize to allow for chemical reactions between various species ²), and to Euler-type equations of motion, which can be written in the very compact form

$$n_X^\sigma \nabla_{[\rho} \pi_{\sigma]}^X = 0, \quad \text{with} \quad \pi_\rho^X \equiv \mu_\rho^X + e^X A_\rho \quad (2)$$

In order to deal with superfluids or superconductors, one must impose the further condition that the momentum covectors π_ρ^X are gradients. This is in fact true on small scales only, because on larger scales the superfluid or the superconductor will be threaded in general by arrays of vortices, due to a global angular momentum (for superfluids) or an external magnetic field (for superconductors). The macroscopic description, which takes into account the average effect of the vortices,

can be done by considering the (generalized) vorticity tensors $w_{\rho\sigma}^x \equiv 2\nabla_{[\rho}\pi_{\sigma]}^x$, now non-zero since there are vorticies, as fundamental quantities ³.

2 Oscillations of superfluid neutron stars

In a neutron star, the existence of a superfluid component, weakly connected to the normal component, leads to a richer spectrum of oscillations. In general relativity, one must take into account not only the matter perturbations but also the perturbations of the metric $g_{\mu\nu}$. In a specific gauge, the even-parity modes can be written (the study of the $m = 0$ modes is enough because of the degeneracy in m) in the form

$$\begin{aligned} \delta g_{00} &= -e^\nu r^l H_0 e^{i\omega t} P_l(\theta), \quad \delta g_{0r} = \delta g_{r0} = -i\omega r^{l+1} H_1 e^{i\omega t} P_l(\theta), \\ \delta g_{rr} &= -e^\lambda r^l H_2 e^{i\omega t} P_l(\theta), \quad \delta g_{\theta\theta} = \delta g_{\phi\phi} / \sin^2 \theta = -r^{l+2} K e^{i\omega t} P_l(\theta), \end{aligned} \quad (3)$$

where P_l stands for the Legendre polynomial of order l . In a two-component model, the matter perturbations can be described by the two matter displacements ξ_n and ξ_p ,

$$\begin{aligned} \xi_n^r &= r^{l-1} e^{-\lambda/2} W_n e^{i\omega t} P_l(\theta), & \xi_n^\theta &= -r^{l-2} V_n e^{i\omega t} \partial_\theta P_l(\theta), \\ \xi_p^r &= r^{l-1} e^{-\lambda/2} W_p e^{i\omega t} P_l(\theta), & \xi_p^\theta &= -r^{l-2} V_p e^{i\omega t} \partial_\theta P_l(\theta). \end{aligned} \quad (4)$$

Inserting the above expressions for the perturbations into the perturbed Einstein's equations as well as the perturbed Euler equations, one ends up with a system of linear equations, consisting of two constraints (one is $H_2 = H_0$, the other expresses H_0 in terms of the other perturbations) and of first order differential equations of the form

$$\frac{dY}{dr} = Q_{l,\omega} Y, \quad (5)$$

where Y is 6-dimensional column vector containing H_1 , K , W_n , V_n , W_p and V_p , and $Q_{l,\omega}$ is a 6×6 matrix with r -dependent coefficients that depend only on the background configuration, as well as on l and ω . Considering the boundary conditions at the center and at the surface of the star, this system can be solved, up to a global amplitude, for *any value* of ω . The physically relevant modes, however, also called quasi-normal modes, correspond the specific values of ω for which the metric outside the star represents *only outgoing gravitational waves*.

A numerical investigation for a very crude model of two independent polytropes (with different adiabatic indices) has shown that the two-component star will exhibit new modes, *superfluid* modes, which are specific of the existence of two components since the two fluids are counter-moving for these modes ⁴.

References

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